Different dispersive waves of bulk elementary excitations in bulk superfluid helium–II at low temperatures

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The Landau's theory of superfluidity of the liquid helium–II at low temperatures is discussed. The Landau's condition of the superfluidity is also discussed, in particular, in the case when the effective phase velocity C_{ph} of the bulk elementary excitations (BEEs) of the liquid becomes zero in both the maxon maximum and the roton minimum. In each energy zones (the phonon, R^- – roton and R^+ – roton branches) there are corresponding two modes of dispersive waves of different types ($C_{ph} > C_g$ and $C_{ph} < C_g$, where C_g is the group velocity of the BEEs), and one corresponding non-dispersive Zakharenko wave. The dispersion relations, for both the free ^{4}He-atom velocities V_{ph}^{at} and V_g^{at}, and the velocities C_{ph} and C_g of the BEEs in the liquid, are drawn for comparison and analyzed. Also, the BEEs' effective masses are plotted in dependence on the wavenumber *k*.
The present paper is written in memory of Acad. L. D. Landau.

1 Introduction

Up to the present time it is well-known that the liquid ^{4}He-helium at low temperatures below the λ – point, T_λ = 2.19 K, gets peculiar properties, the most important of which is superfluidity, which was discovered in 1932 by P.L. Kapitza [1]. Superfluidity is the lack of viscosity during the flow of the liquid helium through slits or thin capillaries. Moreover, the liquid helium exists in liquid state down to absolute zero temperature. These properties of the liquid helium can not be explained by the classical theory and are connected with quantum phenomena; therefore, it is a quantum liquid.

In 1941 L.D. Landau [2] has shown that the superfluidity of the liquid helium at absolute zero is a consequence of the properties of the energy spectra of elementary excitations [3,4,5] (phonons, rotons). He has considered the liquid helium at absolute zero in its normal, unexcited state, which is flowing through a capillary at a constant flow velocity **V**. He has treated phonons and rotons separately, in order to evaluate the liquid flow velocity **V**. He has found that the velocity **V** must be greater than the phonon velocity C_0 in the phonon branch. Namely, he has found C_0 ~ 250 ms^{-1}, more recent value of which is ~ 238 ms^{-1}. Also, the velocity **V** must be greater than the roton velocity in the positive-roton branch. He has concluded that neither phonons nor rotons can be excited, if the flow velocity **V** of the liquid is not too large, and therefore, the liquid flow does not slow down. Therefore, the liquid helium-II discloses the superfluidity phenomenon. The theoretically evaluated minimum velocity C_{01} near the roton minimum of the liquid helium-II can be found in Ref. [5], and the found value of C_{01} is ~ 60 ms^{-1}. There was also discussed that this evaluated value of the roton velocity C_{01} is several orders greater than the one which is experimentally observed.

In the 1980's W.G. Stirling [6,7] has investigated the energy spectra of the bulk elementary excitations in the liquid helium-II by new high-resolution neutron scattering technique. These measurements have clearly shown the so-called "phonon-backflow". He has introduced calculated values of the phase velocities of the elementary excitations of the liquid helium using the measured values of the wavenumbers *k* and the excitations' energies. Unfortunately, not many points were shown for excitation's wavenumbers *k* both below 0.1Å^{-1} and above 0.8Å^{-1}. Moreover, "Not relevant" was written for the excitation's phase velocities for the wavenumbers *k* greater than 1.13Å^{-1}, and this is true. After it, A.C. Forbes and A.F.G. Wyatt [8] have drawn dependence of absolute value of the group velocity C_g of the BEEs on the wavenumber *k*, probably, using the measured data of Refs. [6,7].

In this paper, the dispersion relation of both the group velocity C_g and the effective phase velocity C_{ph} of the bulk elementary excitations are shown for comparison and analysis, as well as dispersion relation for a free ^{4}He-atom that can be useful to help to better understand some quantum effects, such as the "quantum evaporation" [9] of the helium atoms by the corresponding BEEs from the liquid helium surface up to vacuum, the "quantum condensation" of the atoms on the liquid surface, reflection of the corresponding BEEs back to the bulk liquid helium. It could help to better understand the recent works concerning the so-called roton-backflow [10,11].

2 Landau's theory

Let us treat the flow of the liquid helium-II through a capillary at a constant velocity **V** for simplicity as it was done by L D Landau [2], because in this coordinate system the liquid helium–II is at rest, but the capillary walls move with a velocity – **V**. The liquid helium–II must start to move owing to the presence of viscosity. It is obvious that the motion with creation of the elementary excitations of the liquid must begin in boundary layers of the liquid to the walls.

If an elementary excitation is excited in the liquid, the liquid energy E_{liq} is equal to the excitation energy E_{ext}. The energy of the normal state is equal to zero. Therefore, the liquid momentum $\mathbf{P_{liq}}$ is the excitation momentum $\mathbf{P_{ext}}$. It is always treated in the coordinate system in which the liquid was initially at rest. And it is possible now to write that $E_{liq} = E_{ext}$ and $\mathbf{P_{liq}} = \mathbf{P_{ext}}$, where the excitation energy is usually taken as $E_{ext} = C_{ext}(P_{ext})*P_{ext}$ and C_{ext} can be the group velocity of an elementary excitation (phonon, roton). The velocity C_{ext} can be written as $C_{ext} = C_0*\zeta(P_{ext})$, where ζ is a polynomial and the velocity C_0 is taken, in order to have good approximation, which can be different for each branch of the elementary excitations spectra.

In the coordinate system, in which now the capillary is at rest, it is possible to write for the energy E and the momentum $\mathbf{P}$:

$$E = E_{ext} + \mathbf{P_{liq}}\mathbf{V} + MV^2/2 \quad \text{and} \quad \mathbf{P} = \mathbf{P_{liq}} + M\mathbf{V}, \tag{1}$$

where M is the mass of the liquid helium and $MV^2/2$ is the initial kinetic energy of the flowing liquid helium. Also, the energy can be written as

$$E = C_{ext}P_{ext} + \mathbf{P_{ext}}\mathbf{V} + MV^2/2, \tag{2}$$

where $P_{ext} = \mu^* C_{ext}$ and μ^*, C_{ext} are the effective mass and the velocity of an elementary excitation of the liquid helium, respectively. The term ($C_{ext}P_{ext} + \mathbf{P_{ext}}\mathbf{V}$) is the change of the energy due to an elementary excitation. The Landau's suggestion is that this energy change must be negative, because the energy of the flowing liquid must decrease:

$$C_{ext}P_{ext} + \mathbf{P_{ext}}\mathbf{V} < 0. \tag{3}$$

Now it is possible to analyze the expression (3) and to conclude similar to what was done by L.D. Landau: according to Eq. (3), the absolute value of the velocity **V** must be greater than the one of the velocity C_{ext}, in order to fulfill the condition (3):

$$V > C_{ext}. \tag{4}$$

And Landau's conclusion is: at smaller velocities the interaction with the walls of the capillary cannot give rise to creation of an elementary excitation. The problem is to find a minimal value of the velocity C_{ext}. If the velocity of flow in the liquid helium-II is greater than the value of the $C_0 \sim 250$ ms^{-1}, corresponding elementary excitations can be excited in the liquid. When the flow of the liquid does not slow down, the liquid helium–II discloses the phenomenon of superfluidity. However, he has left aside the question: whether the superfluidity disappears at smaller velocities than the found value ~ 60 ms^{-1}, according to Ref. [5]. Also, Landau has treated the effective mass of the bulk elementary excitations in calculations, which is not equal to the free ^{4}He atom mass m_4 and gives better correlation with experiments: $\mu = \alpha m_4$, where α is a suitable factor. In Landau's theory, the superfluid helium-II consists of both normal and superfluid components, treating superfluid motions as potential ones.

3 Dispersions of microscopic and "macroscopic" free quasi-particles

The condition (4) could be fulfilled for each bulk elementary excitation, because each elementary excitation possesses the effective phase velocity C_{ph} and the effective mass μ^*, as well as the group velocity C_g. Both the C_{ph} and the C_g can become zero for some special cases. This occurs both at the maxon-maximum and at the roton-minimum in the energy spectra of bulk elementary excitations in the liquid. The effective phase velocity C_{ph} becomes equal to zero that will be shown below. The group velocity C_g becomes equal to zero that was also shown in Ref. [8]. The energy spectra of possible free quasi-particles are shown in figure 1. On the other hand, the BEEs energy spectra of the liquid helium-II are shown in figure 2. Absolute value of the effective phase velocity C_{ph} can be written near the maxon-maximum as:

$$C_{ph}(k - k_{\Delta m}) = \mathrm{Abs}\left(\frac{1}{\hbar}\frac{E - E_{\Delta m}}{k - k_{\Delta m}}\right) = \mathrm{Abs}\left[\frac{\hbar}{2\mu^*}(k - k_{\Delta m})\right], \tag{5}$$

where μ^* is the effective mass of a bulk elementary excitation, which depends on the wavenumber k, $\mu^* = \mu^*(k - k_{\Delta m})$. The energy $E_{\Delta m} = 13.85$K and the wavenumber $k_{\Delta m} = 1.125$Å^{-1} are the energy and the wavenumber of the maxon-maximum, respectively. It is clearly seen in (5) that at $k = k_{\Delta m}$ the effective phase velocity C_{ph} falls to zero. Absolute value of the C_{ph} can be written near the roton-minimum as:

$$C_{ph}(k-k_{\Delta R}) = \text{Abs}\left(\frac{1}{\hbar}\frac{E-E_{\Delta R}}{k-k_{\Delta R}}\right) = \\ = \text{Abs}\left[\frac{\hbar}{2\mu^*}(k-k_{\Delta R})\right] , \tag{6}$$

where the effective mass μ^* of a bulk elementary excitation depends on the wavenumber k, $\mu^* = \mu^*(k - k_{\Delta R})$. The energy $E_{\Delta R}$ = 8.61K and the wavenumber $k_{\Delta R}$ = 1.925Å^{-1} are the energy and the wavenumber of the roton-minimum, respectively. It is clearly seen that at $k = k_{\Delta R}$ the effective phase velocity C_{ph} falls to zero, too. Therefore, the BEEs energy spectra of the liquid can be divided into three energy zones. The first energy zone is the phonon branch, the second is the R^- – roton branch and the third is the R^+ – roton branch of the bulk elementary excitations.

Let's discuss the dispersion relations of two possible free quasi-particles, which can be both microscopic (for example, atoms) and "macroscopic" ones. The "macroscopic" quasi-particle means that such quasi-particle behaves as a microscopic one, for example, its energy can be described as $E = \pm \hbar\omega$. Figure 1a represents a free quasi-particle with positive kinetic energy, but figure 1b represents a free quasi-particle with negative kinetic energy. It is clearly seen that these two quasi-particles are not identical. The first quasi-particle in figure 1a has dispersion $V_g > V_{ph}$ for positive wavenumbers, while the second quasi-particle in figure 1b has the other dispersion $V_g < V_{ph}$ for positive wavenumbers. On the other hand, if the first quasi-particle has dispersion $V_g < V_{ph}$ for negative wavenumbers, the second has the other possible dispersion $V_g > V_{ph}$ for negative wavenumbers. However, both free quasi-particles have the dispersion $V_g = 2V_{ph}$ to be independently on the sign of the wavenumber k.

The dependence [13] of the group velocity V_g on the phase one V_{ph} can be written as follows:

$$V_g = V_{ph} + (ik)\frac{dV_{ph}}{d(ik)}, \tag{7}$$

where $V_{ph} = -\omega/(ik)$, but $V_g = -d\omega/d(ik)$ with imaginary unity $i = (-1)^{1/2}$. Therefore, for negative values in (7) there is dispersion $V_g < V_{ph}$ for free quasi-particles. Also, classical Love waves in the layered system, consisting of isotropic layer on isotropic substrate, possess such dispersion.

For a free quasi-particle with negative energy there is ($k \longleftrightarrow ik$) that allows observation of such unique free quasi-particle in real space with real wavenumbers k that can be shown by the following equation:

$$E = -\hbar\omega = -\frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 (ik)^2}{2m}. \tag{8}$$

The well-known dependence of negative energy on the group velocity can be shown as follows:

$$E = -\hbar\omega = \frac{m(\pm iV_g)^2}{2} = -\frac{m(\pm V_g)^2}{2}, \tag{9}$$

where both the signs "±i" and "±" relate to the wavenumber k, but not to the frequency ω, in order to keep only real energies. It is thought that + ω and – ω are identical. However, this is not completely so. Negative energies could relate to latent energies. It is well-known that for quasi-particles with small both energies and momenta there is $V_g \sim V_{ph}$ that represents the phonon definition. This occurs in figure 1 for $k \to 0$, but for the Bose-Einstein condensation (BEC) there is $V_g = V_{ph} = 0$. Phonons are also called as sound waves.

4 Dispersion relations

Both the effective phase velocity C_{ph} and the group velocity C_g of the bulk elementary excitations of the liquid helium are shown in figure 2 in dependence on the wavenumber k. The effective phase velocity C_{ph} must become equal to zero that is seen from Eqs. (5) and (6), which describe behavior of the C_{ph} around corresponding energy pits, both at the maxon maximum with $k = k_{\Delta m}$ = 1.125Å^{-1} and at the roton minimum with $k = k_{\Delta R}$ = 1.925Å^{-1}. Also, at k = 0 the effective phase velocity $C_{ph} = C_0$ = 238 ms^{-1} corresponds to the normal, unexcited state of the liquid. The necessity of treating the effective phase velocity C_{ph} instead of the phase velocity V_{ph} for the bulk elementary excitations is introduced, because usage of the V_{ph} in this case is "not relevant" as it was noted by W.G. Stirling [6,7]. However, the experimental data of Refs. [6, 7] are good, in order to calculate the group velocity C_g. Around the energy pits (the maxon maximum and the roton minimum) the energy of corresponding bulk elementary excitation as a quasi-particle can be approximated by the one for a free quasi-particle, $E = \hbar^2 k^2 / 2\mu^*$, where μ^* is the effective mass.

The group velocity C_g depends on the effective phase velocity (5) and (6) in each energy zone as:

$$C_g = C_{ph} + (k-k_\Delta)\frac{dC_{ph}}{d(k-k_\Delta)} \tag{10}$$

and therefore, at boundaries (both at the maxon-maximum and at the roton-minimum) of the energy zones $k = k_\Delta$ and $C_g = C_{ph} = 0$. According to the results obtained in Ref. [13] concerning the dependence of the group velocity on the phase velocity, which is true for (10) as well, there is the following: if the effective phase velocity C_{ph} decreases, the group velocity must be less than the C_{ph}, and if the effective phase velocity C_{ph} increases, the group velocity must be greater than the phase one. Moreover, as it was shown in Ref. [13], once the effective phase velocity has maximum or minimum in dependence on the wavenumber k, at these points, the phase and group velocities are equal, and therefore, at these points it is dealt with the new type of non-dispersive Zakharenko waves [13]. By straightforward analyzing of the energy spectra of the liquid helium it is possible to find that each energy zone (the phonon, R^- – roton and R^+ – roton branches) has only one corresponding non-dispersive Zakharenko wave (the Stirling's experimental data [6,7] give several non-dispersive waves with energy ~ 1K – 2K that is probably incorrect, because there are difficulties to measure in this energy region), and characteristics of which are listed in table 1. As it is seen in figure 2, the corresponding non-dispersive Zakharenko wave divides each energy zone into two energy sub-zones or two modes of dispersive waves with different dispersions ($C_{ph} > C_g$ or $C_{ph} < C_g$).

The energy spectrum of a free ^{4}He-atom positioned in the coordinate beginning (at $E_k^{at}(k = 0) = 0$, but not at $E_k^{at}(k = 0) = E_{bind} = 7.15$K [14,15,16]) is shown in figure 2 by dotted line in dependence on the wavenumber k. It was done by this way, because if there is $E_k^{at}(k = 0) = E_{bind}$, a "forbidden zone" will occur for the atom kinetic energy from $k = 0$ to $k = k_{bind}$ (the wavenumber k is the same that is assumed). It is possible to evaluate k_{bind} from $E_{bind} = \hbar^2 k_{bind}^2 / 2m_4 k_B = 7.15\text{K}$. Hence, k_{bind} can be $k_{bind} = 2m_4 k_B \times 7.15/\hbar^2 \approx 1.09\,\text{Å}^{-1}$, where $k_B = 1.38*10^{-23}$ JK^{-1} is the Bolzmann constant and ħ = $1.05459*10^{-34}$ Js is the Planck constant, and $m_4 = 6.667*10^{-27}$ kg is the ^{4}He-atom mass. The kinetic energy of a free ^{4}He-atom can now be written as:

$$E_k^{at} = \frac{\hbar^2 k^2}{2m_4}. \tag{11}$$

Therefore, the phase velocity of the helium atom is equal to

$$V_{ph}^{at} = \frac{\hbar k}{2m_4} \tag{12}$$

and the group velocity of the atom is

$$V_g^{at} = \frac{\hbar k}{m_4}. \tag{13}$$

For the helium atom, the phase velocity and the group one are shown by two straight lines in figure 2 in dependence on the wavenumber k, beginning at $k = 0$. It is possible now to write that the helium atom represents dispersive waves with the constant relationship $V_g^{at} = 2V_{ph}^{at}$ that is seen from Eqs. (12) and (13). Comparison of the phase and group velocities of the helium atom with the ones of the bulk elementary excitations of the liquid helium, which is a quantum liquid, can throw light on the quantum effects such as both the "quantum evaporation" process and the "quantum condensation" one.

Let us treat the phase velocity, that is used in Physical Acoustics, of a ^{4}He-atom V_{ph}^{at}, but not the atom group velocity V_g^{at}. The V_{ph}^{at} crosses the effective phase velocity C_{ph} of the bulk elementary excitations near the maxon-maximum (see the first energy zone in figure 2). It means that one type of oscillations can excite the other type of oscillations at that crossing point. This effect at crossing points of phase velocities is experimentally difficult-removable in Acoustics of solid layered systems, where for example, Love type waves with polarization perpendicular to the sagittal plane can excite dispersive Rayleigh type waves with polarization in the sagittal plane that was mentioned in Ref. [18]. This is applicable in the present case, too. Therefore, in the first energy zone, an energy leak can occur at the crossing point between the V_{ph}^{at} of a free ^{4}He-atom and the C_{ph} of the bulk elementary excitation (the high-energy phonon) with $C_{ph} = V_{ph}^{at} \sim 75$ ms^{-1} at the wavenumber k near k_{bind}. Thus, these BEEs in the first energy zone can be readily excited by atomic beams consisting of ^{4}He-atoms. In the second energy zone (the R^- – roton branch), the straight line of the atom phase velocity V_{ph}^{at} approaches the maximum absolute value of the effective phase velocity C_{ph}, so the second non-dispersive Zakharenko wave can be created by a ^{4}He-atom (see the energy zone 2 both in figure 2 and in table 1). In the third energy zone (the R^+ – roton branch), the atom phase velocity V_{ph}^{at} approaches the maximum value of the effective phase velocity, too, and therefore, the third non-dispersive Zakharenko wave can be readily excited by a ^{4}He-atom (see the energy zone 3 both in figure 2 and in table 1).

The energies of the helium atoms, which can create both the second non-dispersive Zakharenko waves and the third ones, are given in table 1, respectively. For example, in order to create the third non-dispersive Zakharenko wave with the energy ~ 15.8K, one ^{4}He-atom needs to reach its own kinetic energy ~ 35.6K, because an energy leak can exist between these two different types of oscillations. This atom energy is two times greater than the energy of the third non-dispersive Zakharenko wave that could show coupling between such two BEEs into pairs. The bulk elementary excitations, created by the helium atom beams in the liquid, are experimentally observed. In addition, in the second energy zone in figure 2, the energy of a free ^{4}He-atom E_{at} crosses the corresponding BEEs' energy in the second energy zone (the energy of the second non-dispersive Zakharenko wave) that means they could be equal. Figure 2 supports this.

Figure 3 represents dependence of the effective mass μ^* of the bulk elementary excitations of the liquid helium-II at low temperatures on the wavenumber k. Three points show the effective masses of the bulk elementary excitations, which correspond to the non-dispersive Zakharenko waves, respectively. In the energy zone 1 (the phonon branch), the effective mass μ^* goes to zero at the wavenumber $k \rightarrow 0$. The maximal value of μ^* approaches the free helium-atom mass m_4 at the maxon-maximum, $\mu^* \rightarrow m_4$, and after it goes down to its minimum value at the roton-minimum, $\mu^*_{\Delta R} \rightarrow 0.15\ m_4$ for the wavenumber $k = k_{\Delta R} = 1.925Å^{-1}$, and increases again in the third energy zone (see figure 3). Now it is possible to give, for comparison, the values of $\mu^*_{\Delta R}$, which were obtained by L.D. Landau [4], $\mu^*_{\Delta R} = 0.77\ m_4$, and by R.P. Feynman and M. Cohen [19], $\mu^*_{\Delta R} = 0.4\ m_4$.

The main purpose of the present theoretical work was to show the existence of one corresponding non-dispersive Zakharenko wave in each energy zone. The present theoretical results can be improved in the future, if precise experimental data will be available. The results of the present paper explain the experimental results of the experiments by Wyatt et al. [20] in the new way. The differences between explanations of the experimental results [20] and the same in the present work are summarized in Table 2. It is naturally in Acoustics that non-dispersive waves can propagate for longer distances than dispersive waves.

In addition, Wyatt et al. [21] have shown dependence of the 3*pp*-process angle on the BEEs' energy only for the first energy zone. They have obtained a maximum value of the angle equaled to $\theta \sim 11.2^o$. The angle θ falls to zero at the critical energy $E_c \sim 8.26$K, according to the results of Ref. [21]. However, they did not show the dependence $\theta(E)$ for energies $E > E_c$ in the first energy zone. It is necessary to emphasize that their dependence $\theta(E)$ is valid only qualitatively, but not quantitatively, because their group velocity V_g [8] repeats behavior of the Stirling's phase velocity V_{ph} [6, 7]. It is noted that up to the present there are no more precise measurements of the phase velocity V_{ph} than those carried out by Stirling [6,7]. Particularly, both the group velocity by Wyatt et al. [8] and the phase velocity by Stirling [6,7] have maximum at the same wavenumber. It is obvious that Wyatt et al. have used the Stirling's experimental data, in order to calculate the group velocity. It was shown in the present paper in figure 2 that the group velocity has its minimum at smaller wavenumber than the phase velocity. Therefore, they [21] have obtained the critical energy $E_c(\theta = 0) \sim 8.26$K, but not $E_c \sim 6.2$K that must be according to Stirling's maximum phase velocity. Then, a correct value of the maximum 3*pp*-process angle could be less than 11.2^o relating to energy near maximum group velocity that must be verified. It is also noted that for energies $E > E_c$ there are negative values of the angle θ. However, this matters, because $\cos(\theta)$ is an odd function. All three cases of 3*pp*-scattering threshold are also written in classical and famous textbook [22], see also Refs. [23,24,25]. A correct dependence $\theta(E)$ can be reported by the Author in the future for all three energy zones.

It is also noted that the binding energy $E_b = 7.15$ K by J. Wilks [17] is taken as the chemical potential in all works, see for example Refs. [9,11,26,27]. However, the liquid helium-II consists of both "maxons" and "rotons" representing free quasi-particles at both the maxon maximum and the roton minimum, respectively, but not of free helium atoms ^{4}He. Therefore, it is necessary to take a maxon or roton energy in its corresponding energy zone as chemical potential representing potential energy. The interesting thing is in the second energy zone, where there are both maxon and roton at the energy zone boundaries.

The first non-dispersive Zakharenko wave in the BEEs first energy zone with energy ~ 6.2 K [6,7] is very close to the binding energy $E_b = 7.15$ K [17] representing the helium atom evaporation from the liquid surface due to the crossing point between two phase velocities V_{ph} and c_{ph}, where c_{ph} is the phase velocity of the corresponding surface elementary excitations in the first energy zone. This fact gives a possibility for the corresponding BEEs in the first energy zone to take part in the helium atom evaporation at the liquid surface by some interactions at the liquid surface. The existence of the crossing points in figure 2 between the corresponding BEE phase velocity *Cph* and the helium atom phase velocity *Vph* could mean the existence of a "time-space natural window". One oscillation type representing the corresponding BEE in the liquid helium needs this window at the liquid-vacuum boundary to pass its energy to the other oscillation type, representing the helium atom being also a wave, which can propagate in vacuum. This could mean that the corresponding BEEs, being energy quanta in the liquid helium propagating for long distances in the liquid, continue their propagation in vacuum already becoming the helium atoms, representing energy quanta in vacuum. Hence, it is possible to be sure that there are no probabilities for the quantum evaporation process, as well as for the quantum condensation process.

Indeed, there is also possibility for helium atom condensation in both the maxon maximum and the roton minimum, because it is possible to situate the coordinate beginning for a free helium atom in both the maxon maximum and the roton minimum, where there are trivial wavenumbers for corresponding BEEs. In these cases, an atom condensation does not give the BEE creation. This could also mean the BEE creation with trivial propagating velocity. It is noted that the phonon, negative and positive roton branches were originally situated above each other, but not near each other as shown in figure 2. It is also noted that all BEEs in each branch obey the phonon definition from the view point discussed in the present paper, namely, they all represent weakly dispersive waves with relatively small energies and momenta. Therefore, the phonons, negative and positive rotons could be called as phonons, thermal and supra-thermal phonons, respectively. There is an excellent and classical work [28] by Lord Rayleigh, see also Ref. [29], where he has discussed the relationship d(kV)/dk representing the group velocity coupled with energy. For example, air sound waves are weakly dispersive, therefore they can propagate for long distances, but not too far. In addition, there are works, in which the superfliud helium-II is discussed as normal, but only ultra-cold liquid. Puchkov et al. have recently highlighted such a problem. For this treatment it is possible to emphasize that liquid helium-II possesses the thermo-mechanical effect [30], which is anomalously great in liquid helium-II when compared with other normal liquids.

5 Conclusions

The Landau's theory of superfluidity of the liquid helium at absolute zero developed in the 1940's has shown the condition of the superfluidity: absolute value of the liquid flow velocity **V** must be greater than the velocity C of an elementary excitation, $V > C$. He has only done comparison with the sound velocity $C = C_0 \sim 250$ ms^{-1}, now it is used 238 ms^{-1}, and with the roton velocity (this value from Ref. [5] is $C = C_{01} \sim 60$ ms^{-1}) near the roton minimum. Because each BEE of liquid helium-II possesses its own velocity C_{ext}, the Landau's condition of the superfluidity could be written as $V > C_{ext}$, and as it was shown in the present work, the velocity C_{ext} becomes equal to zero at both the maxon maximum and the roton minimum. Therefore, the condition $V > C_{ext}$ will even be fulfilled for velocity $C_{ph} = C_g = 0$, because it should be $V > 0$ for the BEEs appearance, and therefore, the liquid could be moved by exciting elementary excitations in the liquid with zero velocity.

The energy spectra of the bulk elementary excitations consist of three energy zones (the phonon, R^- – roton and R^+ – roton branches). Each energy zone contains two modes of dispersive waves ("dispersive bulk elementary excitations"), for which the effective phase velocity C_{ph} is unequal to the group velocity C_g, and one corresponding non-dispersive Zakharenko wave, $C_{ph} = C_g \neq 0$ and $dC_{ph}/dk = dC_{ph}/d\omega = 0$. At the boundaries (at both the maxon-maximum and the roton-minimum), which are called the Brillouin zone boundaries, between two neighbor energy zones, both the effective phase velocity C_{ph} and the group velocity C_g become equal to zero. The corresponding non-dispersive Zakharenko waves can be readily experimentally excited both by the helium atom beams, "striking" the liquid surface, and by pulsed heaters in the liquid. Also, high-energy phonons with energies near the maxon energy (13.85K) could be excited by the helium atom beams, because at the crossing point between the effective phase velocity C_{ph} and the atom phase velocity V_{ph}^{at} there can occur an energy leak between these two different types of oscillations.

The results, obtained the in the present work, correlate closely with the experimental results of A.F.G. Wyatt et. al. [20]. However, their explanations of their experimental results are somewhat incorrect. They believe that both the low-energy phonons (~ 1K – 2K, ~ 238 ms^{-1}) and the high-energy phonons (~ 10K, ~ 188 ms^{-1}) are experimentally observed. However, the present results show that both the first non-dispersive Zakharenko wave (the first energy zone or the phonon branch, ~ 6.2K, ~ 249 ms^{-1}) and the third non-dispersive Zakharenko wave (the third energy zone or the R^+ -roton branch, ~ 15K – 16K, ~ 192 ms^{-1}), respectively, are observed.

The Bose-Einstein condensation (BEC) in the liquid helium at the maxon maximum could be such a situation when both the effective phase velocity C_{ph} and the group velocity C_g become equal to zero that could mean propagation of the corresponding BEEs with zero velocity. The bulk elementary excitation (the roton) near the roton minimum behaves as a free quasi-particle with the usual dispersion, $C_g = 2C_{ph}$, for free quasi-particles, while the other bulk elementary excitation (the maxon which is also studied as the BEC [31–34]) near the maxon maximum behaves as a quasi-particle with unique dispersion, $C_{ph} > C_g$, according to the results of the present paper. It is also mentioned that the Landau's superfluidity theory is a macroscopic theory, but not a microscopic one. Also, the BEEs effective masses were plotted in dependence on the wavenumber k. The effective mass goes up from zero at $k = 0$ to its maximum value $\sim m_4$ at the maxon maximum, where m_4 is the ^{4}He-atom mass, and it decreases afterwards down to its minimum value $\sim 0.15m_4$ at the roton minimum.

In the recent work [35], where the new interpretation of photoeffect was reported, it was noted that free electrons emit the photons, whose energy is equal to the binding energy of the electrons in the molecules. A binding energy could be coupled with crossing points of two phase velocities for two different types of oscillations, because an energy leak occurs at those points. The binding energy $E_B = 7.15$ K by J. Wilks [17] relates to the first energy zone in figure 2, which represents the minimum energy, at which there is the helium atom evaporation, and which, probably, depends on temperature of the liquid helium. Each type of the bulk elementary excitations in the corresponding energy zone could possess one corresponding binding energy. This could be shown and discussed in experimental works, which can be reported in the future.

The non-dispersive Zakharenko waves (the Zakharenko condensation or ZC, $C_{ph} = C_g \neq 0$) can be met in suitable energy zones of different solids, see for example in Ref. [36]. Also, "the phenomenon of superconductivity is in many ways akin to the phenomenon of superfluidity" [2]. In addition, it is necessary to mention about the investigations in Ref. [37], where phonons are also observed, and the dependence of the stress on the shear rate is described by a polynomial, similar to the energy spectra of the liquid helium. Investigations of both the BEC and the ZC in solids/liquids/plasmas, as well as of the phenomenon of superconductivity are excluded from the present paper and could be done in collaboration with different research groups. Probably, the BEC is responsible for keeping information about a free quasi-particle, for example, about the effective mass, while the ZC could be a natural product of hybridization occurred everywhere over each energy zone.

The Bose-Einstein condensation with the condition $C_{ph} = C_g = 0$ is met not only in quantum systems at low temperatures, but also in different acoustical systems at room temperatures, for example, Lamb waves in isotropic plates. Both symmetric and anti-symmetric modes can exist in plates (see the famous book [38] by I.A. Viktorov). The lowest-order anti-symmetric mode has linear behavior of the dependence of both the phase velocity V_{ph} and the group velocity V_g on the wavenumber $kh \to 0$ as $V_g = 2V_{ph}$, like the behavior of a free quasi-particle coming to the BEC ($V_{ph} = V_g = 0$) at $kh = 0$.

6 Acknowledgments

I thank Professor G.P. Srivastava for helping me to come in 2000 to the Exeter University, England to study low temperature Physics that was useful for my experience in this research area. Also, I gratefully acknowledge the Referees for useful notes and some corrections of my English usage. In addition, I thank Professor A.F.G. Wyatt for the Stirling's experimental data given me at that time I was in the Exeter University, England, UK.

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Table 1. Characteristics of three non-dispersive Zakharenko waves in bulk superfluid helium–II at low temperatures given in comparison with a free helium atom ^4He, where $k_{\Delta R}$ = 1.925Å^{-1} and $E_{\Delta R}$ = 8.6K.

Energy zone	k, [Å^{-1}]	$\|k-k_{\Delta R}\|$, [Å^{-1}]	E, [K]	$\|E-E_{\Delta R}\|$ [K]	$C_{ph}=C_g$, [ms^{-1}]	μ^*, 10^{-27} [kg]	Free ^{4}He atom, $m_4 = 6.667*10^{-27}$ [kg] E_k^{at}, [K]	V_{ph}^{at}, [ms^{-1}]	V_g^{at}, [ms^{-1}]
1	0.325	–	6.17	–	248.6	1.38	0.64	25.7	51.41
2	1.475	0.45	12.3	3.7	116.6	4.07	13.15	116.7	233.32
3	2.425	0.50	15.8	7.2	191.3	2.76	35.54	191.8	386.59

Table 2. Comparison of different explanations of observed bulk elementary excitations (BEEs).

Experimentally measured BEEs in bulk superfluid helium-II at low temperatures far from a pulsed heater	**Wyatt et al. explanations of the observed BEEs**	**The explanations of the observed BEEs according to the present work**
The bulk elementary excitations with velosity ~ 244–249 ms^{-1} [20]	Low-energy phonons with energies ~ 1–2K [20] in the phonon branch of the BEEs energy spectra	The first non-dispersive Zakharenko wave with energy ~ 6-7K in the phonon branch of the BEEs energy spectra
The bulk elementary excitations with velosity ~ 180–190 ms^{-1} [20]	High-energy phonons with energies ~ 10K [20] in the phonon branch of the BEEs energy spectra	The third non-dispersive Zakharenko wave with energy ~ 15-16K in the positive roton branch of the BEEs energy spectra

Figure 1. Two types of possible free quasi-particles: (a) the free quasi-particle with positive energy; (b) the free quasi-particle with negative energy, where there is the situation $k \to \mathrm{i}k$ and vice versa. The BEC is also shown for $k \to 0$.

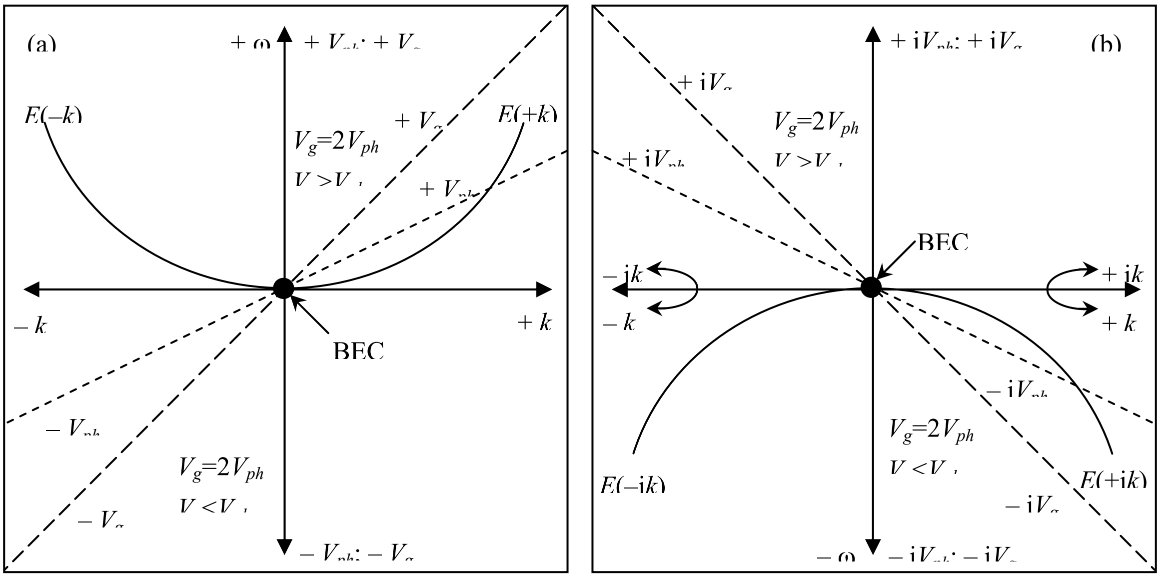

Figure 2. The dispersion relations. Both the energy E_{ext} of the bulk elementary excitations of the liquid helium-II at low temperatures, and the energy E_{at} of a free ^{4}He-atom are shown by point lines in dependence on the wavenumber k. Absolute values of both the effective phase velocity C_{ph} and the group velocity C_g are shown by bold and normal solid lines, respectively. The atom phase velocity V_{ph} and group velocity V_g are shown by two straight lines. Two modes of dispersive waves in each energy zones are shown. Three black points in the energy zones correspond to the non-dispersive Zakharenko waves.

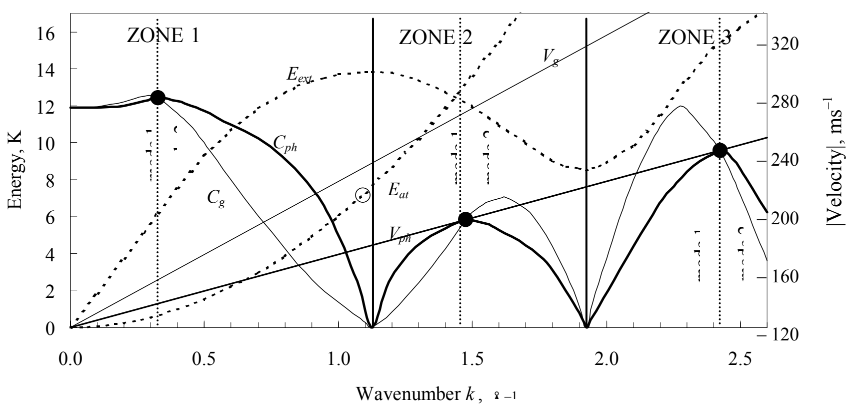

Figure 3. The effective masses $\mu^* = \text{Abs}[\hbar(k - k_\Delta) / C_g]$, 10^{-27} kg of the BEEs of the liquid at low temperatures in dependence on the wavenumber k. The points in three energy zones correspond to the effective masses for the non-dispersive Zakharenko waves.