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Creation evidence of the second non-dispersive Zakharenko wave by helium atomic beams in superfluid helium-II at low temperatures

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Abstract. In this work, the experimental results of the creation of the second non-dispersive Zakharenko wave ($C_{\rm ph} = C_{\rm g} \neq 0$) in the negative roton branch (the so-called second sound) of the bulk elementary excitations (BEEs) energy spectra are introduced. Several BEE signals detected by a bolometer situated in the isotopically pure liquid helium-II at low temperatures ~100 mK are shown, which give evidence of negative roton creation in the liquid by helium atomic beams striking the liquid surface. The negative roton signals were clearly distinguished by the following ways: the negative roton signal created by helium atomic beams appeared earlier than the positive roton signal created by the beams, and presence of both positive and negative roton signals together. It is natural that the negative roton creation by the beams requires the ^{4}He-atom energies ~12 K, while the positive roton creation by the atomic beams requires energies ~35 K. Therefore, successive increase in the heater power resulting in an increase in the ^{4}He-atom energies gives solid evidence that the negative rotons are first created in the liquid by the helium atomic beams.

Keywords. Liquid helium-II; low temperatures; bulk elementary excitations; ^{4}He-atomic beams.

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1. Introduction

According to the recent theory developed in ref. [1], in the so-called quantum condensation process, when a ^{4}He atom with specific energy $E_{\rm a}$ can create a bulk elementary excitation (BEE) at the interface between vacuum and the liquid helium, there is the possibility for ^{4}He atoms to create the second non-dispersive Zakharenko wave (negative roton or second sound). The second sound represents temperature waves without density oscillations. The velocity of the second sound was initially measured by Peshkov [2], which is equal to ~140 m s^{-1} at low temperatures $T \rightarrow 0$ that is, probably, the maximum group velocity $C_{\rm g}$. Both the second and fourth sounds exist in the liquid helium only at low temperatures below the critical temperature $T_\lambda = 2.17$ K (for example, see the excellent book [3]). Below T_λ the liquid helium-II has the roton minimum which disappears together with these two sounds above T_λ [3]. However, the first sound exists both below T_λ and above it, according to ref. [3]. At low temperatures, the velocity of the second sound is $\sim(3)^{1/2}$ times lower than the first sound velocity (see ref. [4]).

The energy spectra of the BEEs in the superfluid helium-II [4,5] represent dispersion relations of dependence of the energy E on the quasi-impulse $p = k\hbar$, where k is the wave number, and $\hbar$ is the Planck's constant. Therefore, the phase velocity C_{ph} of the BEEs can be given by the following relation:

$$\frac{E}{k} = C_{\mathrm{ph}} \frac{E}{\omega}, \tag{1}$$

where ω is the angular frequency. Similarly, for the group velocity C_{g} of the BEEs, the following relation holds good:

$$\frac{\mathrm{d}E}{\mathrm{d}k} = C_{\mathrm{g}} \frac{\mathrm{d}E}{\mathrm{d}\omega}. \tag{2}$$

Therefore, in order to experimentally determine both the phase velocity C_{ph} and the group velocity C_{g} of the BEEs, it is necessary to know dependencies of both $E(k)$ and $E(\omega)$. That could be the answer on the Stirling's 'not relevant' in refs [6,7], because in refs [6,7] the dependence $E(k)$ was only used for calculation of the phase velocities C_{ph} of the BEEs. Note that arbitrary functions $F(k)$ and $F(\omega)$ can be experimentally measured instead of $E(k)$ and $E(\omega)$ in eqs (1) and (2) in order to experimentally evaluate the velocities C_{ph} and C_{g}. It is also thought that it can be readily written $F(k)$ and $F(\omega)$ instead of $F(k,\omega)$ because the dependence $\omega(k)$ (or $k(\omega)$) is assumed to be known. For instance, for a free quasi-particle propagating in vacuum, the function $k(\omega)$ can be obtained from the well-known parabolic dependence $\omega(k) = A_0 k^2$, where the constant A_0 is inversely proportional to the quasi-particle mass.

In addition, in the quantum condensation process there is also the possibility to create the third non-dispersive Zakharenko wave [1] (the positive roton or fourth sound) by the helium atomic beams at the interface between vacuum and the liquid helium at low temperatures, according to a recent theory [1]. However, for the creation of the positive rotons, a ^{4}He atom should reach its own kinetic energy $E_{\mathrm{a}} \sim 35$ K that is three times greater than the kinetic energy required for the creation of the negative rotons ($E_{\mathrm{a}} \sim 12$ K) according to ref. [1]. The creation evidence of the third non-dispersive Zakharenko wave (the positive roton) was initially reported recently in ref. [8], while Brown and Wyatt in ref. [9] reporting positive roton creation have observed negative rotons (the second non-dispersive Zakharenko wave, according to ref. [1]). However, Brown and Wyatt believed that the positive rotons were observed. Their explanations are somewhat incorrect and will be discussed below. The non-dispersive Zakharenko waves can be described by the following formulas:

$$\frac{\mathrm{d}C_{\mathrm{ph}}}{\mathrm{d}k} = C_{\mathrm{g}} \frac{\mathrm{d}C_{\mathrm{ph}}}{\mathrm{d}\omega} = 0, \tag{3}$$

$$\frac{\mathrm{d}C_{\mathrm{ph}}}{\mathrm{d}k} = \frac{1}{k}(C_{\mathrm{g}} - C_{\mathrm{ph}}), \tag{4}$$

$$\frac{\mathrm{d}C_{\mathrm{ph}}}{\mathrm{d}\omega} = \frac{C_{\mathrm{ph}}}{\omega}\left(1 - \frac{C_{\mathrm{ph}}}{C_{\mathrm{g}}}\right). \tag{5}$$

Formula (3) shows that the phase velocity C_{ph} is independent of both the angular frequency ω and the wave number k in the same k–ω domain. It is thought that the non-dispersive Zakharenko waves ($C_{\mathrm{g}} = C_{\mathrm{ph}}$) can exist in any energy branch (mode of dispersive waves) for non-zero group velocity C_{g}. It is expected that the group velocity $C_{\mathrm{g}} = 0$ only for free quasi-particles in vacuum possessing the dispersion $C_{\mathrm{g}} = 2C_{\mathrm{ph}}$, according to ref. [1]. Indeed, formulas (4) and (5) give clearance that eq. (3) is satisfied as soon as the phase and group velocities are equal in dispersion relations.

The results of this paper represent the creation evidence of the second non-dispersive Zakharenko wave (the negative roton) by the helium atomic beams at the interface between vacuum and the liquid helium at low temperatures. Explanations of the experimental results are based on a recent theory [1] and the experimental works [8,10]. It is also noted in [3] that the behaviours of the maximum group velocities of the so-called first, second, and fourth sounds are dependent on temperature below T_λ. The so-called third sound [11] propagates in thin superfluid helium films at low temperatures, where the fourth non-dispersive Zakharenko wave can exist. It is mentioned that the non-dispersive Zakharenko waves were initially discovered in ref. [12]. In this work, the recorded signals were also taken from ref. [13].

2. Experimental configuration and results

In the experiments of this work, ^{4}He atomic beams were ejected into vacuum from the heater H positioned in vacuum above the surface of the superfluid helium-II. The liquid was kept at low temperatures ~60 mK in a small experimental cell with line sizes about several centimetres, using the delusion refrigerator technique. The heater H was positioned at an angle 30° to the surface normal (see figure 1). The distance x_1 between the heater H and the point O in figure 1 is ~25 mm, where ^{4}He atoms will strike the liquid surface. The atoms are travelling ballistically from the heater H down to the liquid surface. The helium atoms ejected by the heater H come through three collimators before they strike at the surface. The collimators were used to minimise surface area, on which the atoms strike the surface with creation of BEEs in the so-called quantum condensation process. The created BEEs at the surface are travelling down through the next collimators in the liquid (see figure 1) down to the bolometer B, with which they can be detected. The bolometer B was positioned at an angle 10° to the surface normal. The distance x_2 between the bolometer B and the point O at the liquid surface is ~7 mm. The bolometer B used in these experiments represents a superconducting Zn thin film detector with work area ~1 mm^2, and the heater H represents an Au thin film pulsed heater. Such types of heaters and bolometers were used previously [8–10,14].

Therefore, the BEEs first arrival time t_0 to the bolometer B arises from the sum of time-of-flight t_1 of the helium atoms from the heater H to the surface point O

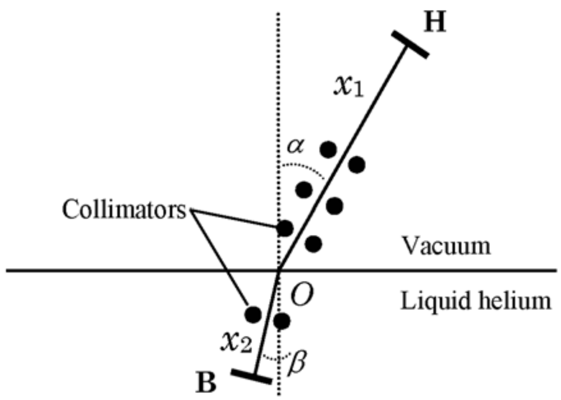

Figure 1. Co-positions of the heater H and the bolometer B. Here the angle α between the heater H and the liquid surface normal is 30° and the distance x_1 is ~25 mm. The angle β between the bolometer B and the normal is 10° and the distance x_2 is ~7 mm.

and time-of-flight t_2 spent by BEEs to travel from the surface point O down to the bolometer B [15]:

$$t_0 = t_1 + t_2 = \frac{x_1}{V_{\rm g}} + \frac{x_2}{C_{\rm g}}, \tag{6}$$

where $V_{\rm g}$ and $C_{\rm g}$ are the group velocities of the helium atoms and created BEEs, respectively, and x_1 and x_2 are the corresponding distances as shown in figure 1. Figure 2 shows recorded signals of BEEs from the bolometer B (figures 2a–e) and another bolometer (figure 2f) used in the previous experiment [8]. In the experimental work [8], the heater was positioned at the same angle 30° to the surface normal with the same distance from the liquid surface that was also used in this experiment, while the other bolometer (from ref. [8]) was positioned at angle 40° to the surface normal with the same distance ~7 mm used for the bolometer B in figure 1. Hence, all six recorded signals of BEEs in figure 2 are suitable for comparison.

The first arrival time of BEEs in figures 2a (the heater power attenuation –23 dB) and 2b (–20 dB) is $\sim(180 \div 200) \times 10^{-6}$ s that is kept constant and difference in figures 2a and 2b is only in amplitudes of the recorded signals. That is natural because more ^{4}He atoms with suitable energies appear with increase in the heater power. Using formula (6) and taking the corresponding atomic group velocity $V_{\rm g} \sim 230$ m s^{-1} from ref. [1], the corresponding group velocity $C_{\rm g}$ of BEEs (the second non-dispersive Zakharenko wave in the negative roton branch of the BEEs spectra) is ~115 m s^{-1} [1]. Therefore, the time-of-flight of suitable ^{4}He atoms can be calculated using $t_1 = x_1/V_{\rm g}$, that is $\sim(25/230) \times 10^{-3} \sim 109 \times 10^{-6}$ s. The time-of-flight of the corresponding negative rotons in the liquid $t_2 = x_2/C_{\rm g}$, that is $\sim(7/115) \times 10^{-3} \sim 61 \times 10^{-6}$ s. Note that it is always assumed zero-condensation time for a ^{4}He atom at the interface between vacuum and the liquid helium [8]. Thus, the total time $t_0 = t_1 + t_2$. In this case, the value of $t_0 \sim 170 \times 10^{-6}$ s is in agreement with the values of $t_0 \sim (180 \div 200) \times 10^{-6}$ s taken from figures 2a

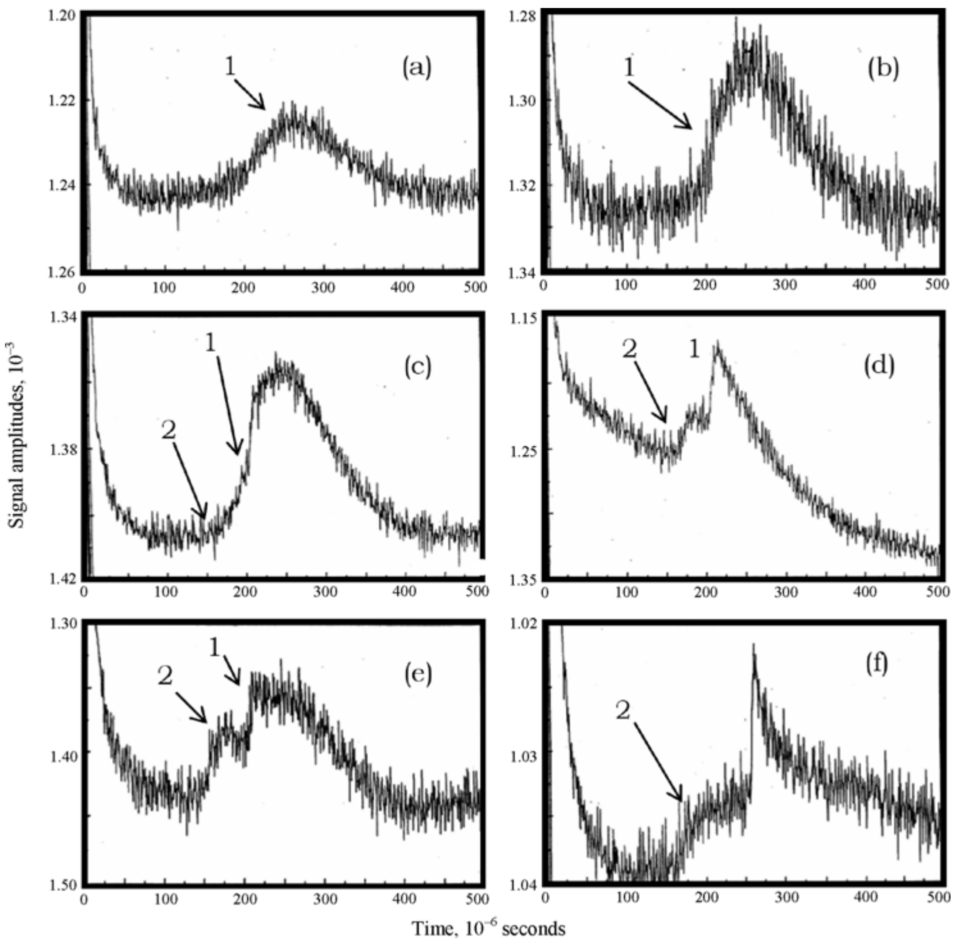

Figure 2. Several detected signals from ref. [13] (symbols 1 and 2 are for the R^-- and the R^+-rotons, respectively) with the heater pulse length 2 μs and the following power attenuations: (**a**) – 26 dB; (**b**) – 23 dB; (**c**) – 20 dB; (**d**) – 13 dB; (**e**) – 8 dB; (**f**) – 20 dB (this signal is from another bolometer situated at an angle 45° and distance ∼7 mm in the liquid with the same heater that was studied in ref. [8]).

and 2b. It is necessary to notify that accuracy of such experiments depends on distance gone by both the helium atoms with suitable energies $E_{\mathrm{a}} \sim 12$ K [1] and the negative rotons, i.e. the second non-dispersive Zakharenko waves. It is assumed that such non-dispersive waves can travel longer distances than dispersive negative rotons with maximum group velocity $\sim 140 \div 160$ m s^{-1} [1]. The corresponding group velocities of the suitable helium atoms with the relationship between the phase and group velocities $V_{\mathrm{g}} = 2V_{\mathrm{ph}}$ are greater than those of created negative and positive rotons, according to ref. [1].

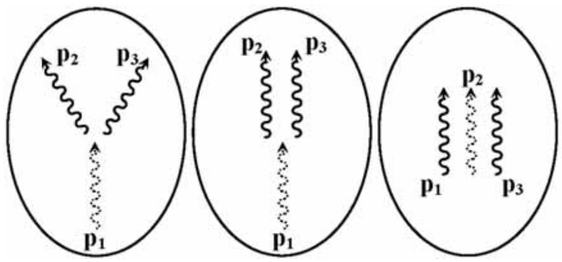

Figure 3. The Feynman diagrams for the three-phonon–phonon process: (**a**) dispersive waves; (**b**) non-dispersive waves; (**c**) the other possibility for non-dispersive waves.

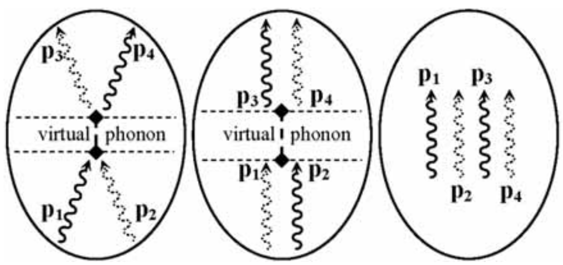

Figure 4. The Feynman diagrams for the four-phonon–phonon process: (**a**) dispersive waves; (**b**) non-dispersive waves; (**c**) the other possibility for non-dispersive waves.

In the experiment carried out by Brown and Wyatt in ref. [9], the corresponding negative rotons (the second non-dispersive Zakharenko wave [1]) are observed. However, they believed that positive rotons were observed. They did not reach the suitable atomic energies $E_{\mathrm{a}} \sim 35$ K [1] required to create the third non-dispersive Zakharenko wave in the positive roton branch of the BEEs energy spectra, because in ref. [9] there is only one roton peak (single roton signal) of BEEs corresponding to the negative rotons with the suitable energies $E_{\mathrm{a}} \sim 12$ K, according to ref. [1]. However, there should be two peaks in the case of the creation of the corresponding positive and negative rotons in such experiments. In addition to the first roton signal, the second roton signal must appear once the suitable atomic energies $E_{\mathrm{a}} \sim 35$ K are reached for the creation of the corresponding positive rotons. It is thought that the created helium atomic beams in such experiments have a kinetic energy distribution for the atoms, and slow and fast ^{4}He atoms must always be in a wide range of atomic energies of ^{4}He atoms created by pulsed heaters. For the creation of suitable negative rotons, corresponding atomic energies $E_{\mathrm{a}} \sim 12$ K are required that is three times smaller than the energy values of $E_{\mathrm{a}} \sim 35$ K required for the creation of suitable positive rotons (the third non-dispersive Zakharenko wave).

On the other hand, the experimental configuration used in ref. [9], $x_1 \sim 6.5$ mm and $x_2 \sim 6.6$ mm, is also not good: it is necessary to do experiments with the distances x_1 and x_2 as far as possible from the liquid surface. That was an additional difficulty for their explanations of their results [9]. In the experimental work carried out by Edwards *et al* [16], the used distance was $L \sim 80$ mm, but they did not mention which rotons were created in their experiments. Also, in ref. [16], angles were discussed, at which maximum intensities of created BEEs were observed. That is natural, because the helium atoms and the created BEEs have different dispersions. In addition, a heater with a fixed angle to the surface normal ejects the helium atoms, which can strike the liquid helium surface at arbitrary angles owing to ripplons at the surface. Therefore, the results of ref. [16] can also give information about curvature of the liquid surface.

The experiments carried out by Zakharenko [8] recorded ^{4}He atomic signals without the liquid in the experimental cell, i.e. with very small amount of liquid that is enough to cover both the bolometer in ref. [8] and the heater with ^{4}He thin film. The atomic signals [8] were detected with the other bolometer situated at an angle 40° to the surface normal. For the heater power –23 dB [8], there were already fastest atoms at the first arrival time with energies $E_{\mathrm{a}} \sim 12$ K required for the creation of the corresponding negative rotons. That is well-correlated with negative roton signals shown in figures 2a (–23 dB) and 2b (–20 dB) as discussed above. For the creation of the corresponding positive rotons, suitable atomic energies $E_{\mathrm{a}} \sim 35$ K [1] are required. Figure 2f (–20 dB) shows detected signals from the other bolometer (angle 40°) [8], from which it is seen that there is very weak signal slightly above the noisy level with the first arrival time $\sim$150 μs to 160 μs. The weak signal can correspond to the third non-dispersive Zakharenko wave in the positive roton branch, because there cannot exist many fastest atoms with the suitable energies that was discussed in ref. [8]. Atomic signals from the bolometer B without the liquid were not observed in this experimental work, probably, due to co-position between the heater H and the bolometer B to the surface normal and the bolometer collimators. The position of the other bolometer in the work of Zakharenko [8] allowed him to observe atomic signals. In the work of Brown and Wyatt [9], an atomic signal was not shown.

There are already two detected signals of BEEs in figures 2c (–18 dB), 2d (–13 dB), and 2e (–8 dB). The first arrival time of the first BEE signal (symbol 2 in the figures) is $\sim$140 μs to 150 μs. In figure 2c there is no clear maximum of the first BEE signal, and therefore, the two BEE signals (symbols 1 and 2) look like one signal, because the first BEE signal is very weak for the heater power -18 dB in figure 2c compared with the second large BEE signal (the negative rotons). In figures 2d and 2e, already two peaks of these BEEs' signals are clearly seen due to increasing heater power. The first BEE signal denoted by symbol 1 relates to the BEE signals shown in figures 2a and 2b (the second non-dispersive Zakharenko wave in the negative roton branch). The second weaker BEE signal denoted by the symbol 2 relates to the third non-dispersive Zakharenko wave in the positive roton branch, for which the helium atoms must have suitable energies $E_{\mathrm{a}} \sim 35$ K and the group velocities $V_{\mathrm{g}} \sim 380$ m s^{-1} [1] for the creation of the positive rotons. The created suitable positive rotons must have energies $\sim$16 K to 17 K with the group velocities $C_{\mathrm{g}} \sim 180$ m s^{-1} to 190 m s^{-1} [1]. It is necessary to emphasise that Brown and Wyatt [9] observed the negative rotons discussed in this work, but not the positive roton that they in ref. [9] believed. Probably, they did not reach the suitable energy $E_{\mathrm{a}} \sim 35$ K for the helium atoms, in order to create suitable positive rotons. Moreover, it was written in ref. [9] that it is impossible to create negative rotons by helium atomic beams.

3. The three-phonon–phonon and four-phonon–phonon processes

It is also possible to discuss that the suitable positive rotons (the third non-dispersive Zakharenko wave) could be coupled with the famous four-phonon–phonon process (4pp), because one suitable helium atom (∼35 K) can create two suitable positive rotons (the Cooper pair) with energy values (∼16 K to 17 K) being approximately two times smaller than those of the atomic energies. On the other hand, the suitable negative rotons (the second non-dispersive Zakharenko wave) could be coupled with the famous three-phonon–phonon process (3pp), because one suitable helium atom (∼12 K) can create one suitable negative roton with approximately the same energy. It is noted that in the BEE phonon branch, the creation of high-energy phonons with suitable energies $E_{\mathrm{ph}} = E_{\mathrm{B}} = 7.15$ K (E_{B} is the binding energy [17]) by the helium atoms must also require suitable atomic energies at the crossing point of the phase velocities V_{ph} and C_{ph} in the first energy zone [1], at which an energy leak occurs (as discussed in refs [1,8]). Therefore, both the quantum condensation process and the quantum evaporation process are inverse processes to each other in both second and third energy zones [1,8] of the BEE energy spectra, but not in the first.

It is noted that solids are also investigated at low temperatures, where heat phonons can propagate (see refs [18–20]). They can also be created by a thin film of 1 mm^2 square heated by the short pulses ∼10 ns to 100 ns. The thin film is situated on a crystal surface. Also, there are measurements with scattering method of slow neutrons, for instance, in NaFl_7 [20], where there are both optical and acoustical phonon branches. Probably, it is possible to write 'heat phonons' ('thermal phonons' or 'resistive phonons') instead of 'negative rotons' in the liquid helium-II.

It is also possible to theoretically evaluate the effective masses of the quasi-particles, which participate in the three-phonon–phonon and four-phonon–phonon processes evaporating the helium atoms. In the negative roton branch of the BEE energy spectra, it is obvious that quasi-particles (thermal phonons) with total energy $E_{\mathrm{total}} \sim 12$ K can evaporate single helium atom with the same energy $E_{\mathrm{a}} \sim 12$ K. Thus energy conservation law is satisfied. According to the BEE energy spectra (see figure 2 of ref. [1]), the bulk quasi-particle energy E_{BEE} is combined from both kinetic (E_{k}) and potential (E_{p}) energies. There are two possibilities for both the kinetic and potential energies of quasi-particles in the negative roton energy branch in order to give BEE energy for a single quasi-particle of the 3pp-process:

$$E_{\mathrm{BEE}} = \mu^* C_{\mathrm{g}}^2/2. \tag{7}$$

The first possibility represents summation of the potential energy ($E_{\Delta\mathrm{R}} \sim 8.6$ K) of the roton minimum with positive kinetic energy E_{k1}:

$$E_{\mathrm{total}} = E_{\Delta\mathrm{R}} + E_{\mathrm{k1}} = \sum (E_{\mathrm{BEE}})_i \sim 12 \text{ K} \tag{8}$$

using the neutron scattering experimental data, and the second possibility is obtained by sticking together the potential energy ($E_{\Delta\mathrm{m}} \sim 13.85$ K) of the maxon maximum with the negative kinetic energy E_{k2}:

$$E_{\mathrm{total}} = E_{\Delta\mathrm{m}} + E_{\mathrm{k2}} = \sum (E_{\mathrm{BEE}})_i \sim 12 \text{ K}. \tag{9}$$

In the superfluidity theory, it is suggested that superfluid motion of the liquid represents a potential motion, which could be coupled with some oscillations within a ^{4}He atom (neutrons and proton–electron pairs), because the helium atoms are in a condensed state in the liquid helium-II, but not free ^{4}He atoms. Therefore, it is assumed that effective mass of a single quasi-particle cannot be greater than the ^{4}He atom mass M_4. It is also assumed that it is possible to treat some potential E_{p} and kinetic E_{k} energies instead of the energies $E_{\Delta\mathrm{R}}$ with E_{k1} and $E_{\Delta\mathrm{m}}$ with E_{k2} because

$$E_{\Delta\mathrm{R}} + E_{\mathrm{k1}} = E_{\Delta\mathrm{m}} + E_{\mathrm{k2}} = E_{\mathrm{p}} + E_{\mathrm{k}} \tag{10}$$

requiring the equality $E_{\mathrm{p}} = E_{\mathrm{k}}$. It is thought that it is natural to take the kinetic energy E_{k} in view of the de Broglie wave:

$$E_{\mathrm{k}} = \hbar^2 k^2/2M^* = M^* C_{\mathrm{g}}^2/2. \tag{11}$$

Using the equality $E_{\mathrm{p}} = E_{\mathrm{k}}$, it is possible to write the total energy as

$$E_{\mathrm{total}} = E_{\mathrm{p}} + E_{\mathrm{k}} = M^* C_{\mathrm{g}}^2. \tag{12}$$

On the other hand, the total energy E_{total} is equal to the energy E_{a} of an evaporated ^{4}He atom

$$E_{\mathrm{total}} = E_{\mathrm{a}} = M_4 V_{\mathrm{g}}^2/2 \sim 12\ \mathrm{K} \quad \text{with } E_{\mathrm{total}} = M^* V_{\mathrm{g}}^2/4. \tag{13}$$

According to a recent theory [1], there is the following relationship:

$$C_{\mathrm{ph}} = C_{\mathrm{g}} = V_{\mathrm{ph}} = V_{\mathrm{g}}/2 \tag{14}$$

for the suitable quasi-particles representing the second non-dispersive Zakharenko waves. Therefore, it is possible to get a relationship between the quasi-particle effective mass μ^* and the helium atom mass M_4: $\mu^* = M^*/3 = 2M_4/3$, because three quasi-particles participate in the 3pp-process and they should be alike. This result is well-correlated with the previous evaluation of the effective mass $\mu^* \sim 4.1\times10^{-27}$ kg for the second non-dispersive Zakharenko waves obtained in ref. [1]. The effective mass μ^* from ref. [1] is only 10% less than the results of this evaluation done above. Some possible Feynman diagrams for the three-phonon–phonon process are shown in figure 3.

It is possible to discuss the helium atom evaporation by suitable BEEs in the positive roton branch of the BEE energy spectra by the same way. However, here the energy E_{a} of an evaporated ^{4}He atom is equal to $E_{\mathrm{a}} = M_4 V_{\mathrm{g}}^2/2 \sim 35$ K that is approximately two times greater than the energy $\sim$17 K of the suitable BEE quasi-particles. Therefore, there is $2E_{\mathrm{total}} = E_{\mathrm{a}} = M_4 V_{\mathrm{g}}^2/2 \sim 35$ K according to the energy conservation law that allow us to state that such BEE quasi-particles are coupled into pairs representing the Cooper pairs. It is noted that the Cooper pairing phenomenon was first discovered in the well-known classical superconductivity theory, in which two electrons each with negative charge are coupled into pairs. Indeed, there is also the relationship of eq. (14) for the suitable quasi-particles representing the third non-dispersive Zakharenko waves, according to the recent theory

[1]. The equality between the energy $2E_{\rm total}$ and the energy $E_{\rm a}$ of an evaporated ^{4}He atom, $2E_{\rm total} = E_{\rm a} = M_4 V_{\rm g}^2/2 \sim 35$ K to 36 K with $2E_{\rm total} = 2M^* V_{\rm g}^2/4$, results in the following relationship between the quasi-particle effective mass μ^* and the helium atom mass M_4: $\mu^* = M_4/2$. It is noted that this evaluated value of the BEE quasi-particle effective mass is also well-correlated with the previous evaluation introduced in ref. [1], using the neutron scattering experimental data. The effective mass value of ref. [1] $\mu^* \sim 2.8 \times 10^{-27}$ kg is approximately 20% less than the results for the effective mass evaluation followed in this paper. It was noted in ref. [1] that the results can be improved in the future, if improved experimental data will be available. Some possible Feynman diagrams for the four-phonon–phonon process are shown in figure 4. The effective masses for both the three-phonon–phonon (3pp) and four-phonon–phonon (4pp) processes are listed in table 1. In addition, it is thought that the 4pp-process can be treated as a double 3pp-process, because by comparing figures 3a with 4a, it is clearly seen that the virtual phonon in figure 4a for the 4pp-process is common for two 3pp-processes. Therefore, both 3pp-processes are coupled through the virtual phonon, and hence, they have an exchange.

Probably, it is natural to treat the liquid helium-II as a liquid consisting of condensed ^{4}He atoms, neutron and proton–electron pairs which are somewhat coupled. Therefore, BEE effective masses must be smaller than the mass of a free ^{4}He atom M_4. It is noted that linear size of the helium atom is approximately equal to 1 Å $= 10^{-10}$ m corresponding to the wavelength $\lambda_{\rm a}$ and wave number $k_{\rm a} = 2\pi/\lambda_{\rm a} \sim 1$ Å^{-1}. However, for such quantum liquid as the liquid helium-II, the maxon maximum wave number $k_{\Delta \rm m} \sim 1.13$ Å^{-1} is greater than the wave number $k_{\rm a}$. Therefore, BEE quasi-particle wavelengths in both negative and positive roton branches are smaller than the wavelength $\lambda_{\rm a}$. For example, $\lambda_{\rm a}$ is approximately 1.5 times greater than the wavelength corresponding to the second non-dispersive Zakharenko wave in the negative roton branch and 2–2.5 times greater than the wavelength corresponding to the third non-dispersive Zakharenko wave in the positive roton branch of the BEE energy spectra. Thus, it is possible to suggest that the liquid helium-II surface in such low temperature experiments at bulk liquid temperatures below $\sim$100 mK represents a smooth surface for such suitable thermal and supra-thermal BEE quasi-particles, which can evaporate ^{4}He atoms.

Indeed, there are difficulties to excite and to measure second sound in the liquid helium-II. However, there is some progress in measurement technique developments for complex investigations of the second sound. For example, a new method was demonstrated in ref. [21] concerning creation of collective excitations in a Bose–Einstein condensed cloud and the speed of sound was determined as a function of condensate density. This method allows comparison with sound propagation in liquid helium-II and is promising for studying higher-lying excitations and perhaps for studying the second sound. In addition, there is a possibility to parametrically generate the second sound. Reference [22] reports the first experimental observation of parametric generation of second sound waves by first sound waves in the superfluid helium-II in a rather narrow temperature range close to the superfluid transition temperature T_λ. It is noted that parametric generation of waves is observed in a wide class of non-linear media. Few examples from refs [23–25] are spin waves in ferrites and anti-ferromagnets and Langmuir waves in plasma parametrically driven

Table 1. Evaluated effective masses of both the second non-dispersive Zakharenko wave (m_{Z2}) in the three-phonon–phonon process (3pp) and the third non-dispersive Zakharenko wave (m_{Z3}) in the four-phonon–phonon process (4pp) with relationships to the ^{4}He-atomic mass ($M_4 = 6.667 \times 10^{-27}$ kg).

Phonon process	Effective mass (10^{-27} kg)	Relationship to M_4
m_{Z2} (3pp)	~ 4.4447	$2M_4/3$
m_{Z3} (4pp)	~ 3.3335	$M_4/2$

by a microwave field, ferrofluid surface waves subjected to an AC tangential magnetic field, and surface waves in liquid dielectrics parametrically excited by an AC electric field. It is well-known that there are two kinds of three-phonon–phonon processes (3pp) in the superfluid helium-II that are responsible for second sound wave generation by first sound waves considered theoretically in refs [26,27]:

- Cherenkov emission representing a first sound phonon decay into a pair of first sound and second sound phonons;
- parametric decay representing a first sound phonon decay into a pair of second sound phonons (thermal phonons).

Both processes result in decay instabilities which are characterized by thresholds in the first sound amplitude. It is thought that the second sound generation at the liquid–vacuum interface by helium atomic beams is the simplest way.

4. Conclusions

In this paper, the possibility in low temperature experiments to excite the second non-dispersive Zakharenko wave (the negative roton or second sound) by the helium atomic beams at the interface between vacuum and the liquid helium was shown. It was also shown and discussed that the creation of the negative rotons by the helium atomic beams is easier than that of the positive rotons, for which much more greater ^{4}He atom energies E_a are required ($E_a \sim 12$ K for the negative roton creation, but $E_a \sim 35$ K for the positive roton creation). The negative roton signals are readily distinguished from the positive roton signals in all experiments by their first arrival time detected by a bolometer and the required corresponding atom velocities for the creation of both negative and positive rotons, using the recent theory [1]. In order to create the negative rotons, the group velocity V_g of the ^{4}He atom is required to be equal to $\sim$230 m s^{-1}, while the required V_g for the creation of the positive rotons is equal to $\sim$380 m s^{-1}, according to ref. [1]. It was also concluded that it is natural that there is difference in angles of the heater H and the bolometer B co-positions to the liquid surface normal for detection of created BEEs in the liquid helium-II. This is due to the fact shown in ref. [1] that the ^{4}He atoms are dispersive waves with constant relationship $C_g = 2C_{ph}$ for the de Broglie waves, which can create the second and third non-dispersive Zakharenko waves (the negative and positive rotons, respectively).

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